

Effects of polarization in electromagnetic processes in oriented crystals at high energy

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Abstract

Under quite generic assumptions the general expression is derived for the probability of circularly polarized photon emission from the longitudinally polarized electron and for the probability of pair creation of longitudinally polarized electron (positron) by circularly polarized photon in oriented crystal in a frame of the quasiclassical operator method. For small angle of incidence the expression turns into constant field limit with corrections due to inhomogeneous character of field in crystal. For relatively large angle of incidence the expression gets over into the theory of coherent radiation or pair creation. It is shown that the crystal is a very effective device for helicity transfer from an electron to photon and back from a photon to electron or positron.

1 Introduction

The study of processes with participation of polarized electrons and photons permits to obtain important physical information. Because of this experiments with use of polarized particles are performed and are planning in many laboratories (BINP, CERN, SLAC, Jefferson Natl Accl Fac, etc). In this paper it is shown that oriented crystal is a unique tool for work with polarized electrons and photons

The quasiclassical operator method developed by authors [1]-[3] is adequate for consideration of the electromagnetic processes at high energy. The probability of photon emission has a form (see [4], p.63, Eq.(2.27); the method is given also in [5],[6])

$$dw = \frac{e^2}{(2\pi)^2} \frac{d^3k}{\omega} \int dt_2 \int dt_1 R^*(t_2) R(t_1) \exp \left[-\frac{i\varepsilon}{\varepsilon'} (kx_2 - kx_1) \right], \quad (1)$$

where $k^\mu = (\omega, \mathbf{k})$ is the 4-momentum of the emitted photon, $k^2 = 0$, $x^\mu(t) = (t, \mathbf{r}(t))$, $x_{1,2} \equiv x(t_{1,2})$, t is the time, and $\mathbf{r}(t)$ is the particle location on a classical trajectory, $kx(t) = \omega t - \mathbf{k}\mathbf{r}(t)$, ε is the energy of initial electron, $\varepsilon' = \varepsilon - \omega$, we employ units such that $\hbar = c = 1$. The matrix element $R(t)$ is defined by the structure of a current. For an electron (spin 1/2 particle) one has

$$\begin{aligned} R(t) &= \frac{m}{\sqrt{\varepsilon\varepsilon'}} \bar{u}_{s_f}(\mathbf{p}') \hat{e}^* u_{s_i}(\mathbf{p}) = \varphi_{\zeta_f}^+ (A(t) + i\boldsymbol{\sigma}\mathbf{B}(t)) \varphi_{\zeta_i}, \\ A(t) &= \frac{1}{2} \left(1 + \frac{\varepsilon}{\varepsilon'} \right) \mathbf{e}^* \boldsymbol{\vartheta}(t), \\ \mathbf{B}(t) &= \frac{\omega}{2\varepsilon'} \left(\mathbf{e}^* \times \left(\frac{\mathbf{n}}{\gamma} - \boldsymbol{\vartheta}(t) \right) \right), \end{aligned} \quad (2)$$

here \mathbf{e} is the vector of the polarization of a photon (the Coulomb gauge is used), the four-component spinors u_{s_f}, u_{s_i} describe the initial (s_i) and final (s_f) polarization of the electron and we use for the description of electron polarization the vector $\boldsymbol{\zeta}$ describing the polarization of the electron (in its rest frame), $\boldsymbol{\zeta}_i$ is the spin vector of initial electron, $\boldsymbol{\zeta}_f$ is the spin vector of final electron, the two-component spinors $\varphi_{\zeta_i}, \varphi_{\zeta_f}$ describe the initial and final polarization of the electron, $\mathbf{v} = \mathbf{v}(t)$ is the electron velocity, $\boldsymbol{\vartheta}(t) = (\mathbf{v} - \mathbf{n}) \simeq \mathbf{v}_\perp(t)$, \mathbf{v}_\perp is the component of particle velocity perpendicular to the vector $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$, $\gamma = \varepsilon/m$ is the Lorentz factor. The expressions in Eq.(2) are given for radiation of ultrarelativistic electrons, they are written down with relativistic accuracy (terms $\sim 1/\gamma$ are neglected) and in the small angle approximation.

The important parameter χ characterizes the quantum effects in an external field, when $\chi \ll 1$ we are in the classical domain and with $\chi \geq 1$ we are already well inside the quantum domain while for pair creation the corresponding parameter is κ

$$\chi = \frac{|\mathbf{F}|\varepsilon}{F_0 m}, \quad \kappa = \frac{|\mathbf{F}|\omega}{F_0 m}, \quad \mathbf{F} = \mathbf{E}_\perp + (\mathbf{v} \times \mathbf{H}), \quad \mathbf{E}_\perp = \mathbf{E} - \mathbf{v}(\mathbf{v}\mathbf{E}), \quad (3)$$

where $\mathbf{E}(\mathbf{H})$ is an electric (magnetic) field, $F_0 = m^2/e = (m^2 c^2 / e \hbar)$ is the quantum boundary (Schwinger) field: $H_0 = 4.41 \cdot 10^{13} \text{Oe}$, $E_0 = 1.32 \cdot 10^{16} \text{V/cm}$.

The quasiclassical operator method is applicable when $H \ll H_0$, $E \ll E_0$ and $\gamma \gg 1$.

Summing the combination $R^*(t_2)R(t_1) = R_2^* R_1$ over final spin states we have

$$\sum_{\zeta_f} R_2^* R_1 = A_2^* A_1 + \mathbf{B}_2^* \mathbf{B}_1 + i [A_2^* (\boldsymbol{\zeta}_i \mathbf{B}_1) - A_1 (\boldsymbol{\zeta}_i \mathbf{B}_2^*) + \boldsymbol{\zeta}_i (\mathbf{B}_2^* \times \mathbf{B}_1)], \quad (4)$$

where the two first terms describe the radiation of unpolarized electrons and the last terms is an addition dependent on the initial spin.

For the longitudinally polarized initial electron and for circular polarization of emitted photon we have [10]

$$\sum_{\zeta_f} R_2^* R_1 = \frac{1}{4\varepsilon'^2} \left\{ \frac{\omega^2}{\gamma^2} (1 + \xi) + [(1 + \xi)\varepsilon^2 + (1 - \xi)\varepsilon'^2] \boldsymbol{\vartheta}_1 \boldsymbol{\vartheta}_2 \right\}, \quad (5)$$

where $\xi = \lambda\zeta$, $\lambda = \pm 1$ is the helicity of emitted photon, $\zeta = \pm 1$ is the helicity of the initial electron. In this expression we omit the terms which vanish after integration over angles of emitted photon.

The probability of polarized pair creation by a circularly polarized photon can be found from Eqs.(1), (2) using standard substitutions:

$$\begin{aligned} \varepsilon' &\rightarrow \varepsilon', & \varepsilon &\rightarrow -\varepsilon, & \omega &\rightarrow -\omega, & \lambda &\rightarrow -\lambda, \\ \zeta_i &\equiv \zeta \rightarrow -\zeta', & \zeta_f &\equiv \zeta' \rightarrow \zeta, & \xi &\rightarrow \xi, & \xi' &\rightarrow -\xi'. \end{aligned} \quad (6)$$

Performing the substitutions Eq.(6) in Eq.(5) the combination for creation of pair with polarized positron by circularly polarized photon

$$\sum_{\xi'} R_2 R_1^* = \frac{m^2}{8\varepsilon^2\varepsilon'^2} \left\{ \omega^2(1 + \xi) + \gamma^2 \boldsymbol{\vartheta}_2 \boldsymbol{\vartheta}_1 \left[\varepsilon^2(1 + \xi) + \varepsilon'^2(1 - \xi) \right] \right\}. \quad (7)$$

It should be noted that a few different spin correlations are known in an external field. But after averaging over directions of crystal field only the longitudinal polarization considered here survives.

2 General approach to electromagnetic processes in oriented crystals

The theory of high-energy electron radiation and electron-positron pair creation in oriented crystals was developed in [7]-[8], and given in [4]. In these publications the radiation from unpolarized electrons was considered including the polarization density matrix of emitted photons. Since Eqs.(5), (7) have the same structure as for unpolarized particles, below we use systematically the methods of mentioned papers to obtain the characteristics of radiation and pair creation for longitudinally polarized particles.

Let us remind that along with the parameter χ which characterizes the quantum properties of radiation there is another parameter

$$\varrho = 2\gamma^2 \langle (\Delta \mathbf{v})^2 \rangle, \quad (8)$$

where $\langle (\Delta \mathbf{v})^2 \rangle = \langle \mathbf{v}^2 \rangle - \langle \mathbf{v} \rangle^2$ and $\langle \dots \rangle$ denotes averaging over time. In the case $\varrho \ll 1$ the radiation is of a dipole nature and it is formed during the time of the order of the period of motion. In the case $\varrho \gg 1$ the radiation is of magnetic bremsstrahlung nature and it is emitted from a small part of the trajectory.

In a crystal the parameter ϱ depends on the angle of incidence ϑ_0 which is the angle between an axis (a plane) of crystal and the momentum of a particle. If $\vartheta_0 \leq \vartheta_c$ (where $\vartheta_c \equiv (2V_0/\varepsilon)^{1/2}$, V_0 is the scale of continuous potential of an axis or a plane relative to which the angle ϑ_0 is defined) electrons falling on a crystal are captured into channels or low above-barrier states, whereas for

$\vartheta_0 \gg \vartheta_c$ the incident particles move high above the barrier. In later case we can describe the motion using the approximation of the rectilinear trajectory, for which we find from Eq.(8) the following estimate $\varrho(\vartheta_0) = (2V_0/m\vartheta_0)^2$. For angles of incidence in the range $\vartheta_0 \leq \vartheta_c$ the transverse (relative to an axis or a plane) velocity of particle is $v_\perp \sim \vartheta_c$ and the parameter obeys $\varrho \sim \varrho_c$ where $\varrho_c = 2V_0\varepsilon/m^2$. This means that side by side with the Lindhard angle ϑ_c the problem under consideration has another characteristic angle $\vartheta_v = V_0/m$ and $\varrho_c = (2\vartheta_v/\vartheta_c)^2$.

We consider here the photon emission (or pair creation) in a thin crystal when the condition $\varrho_c \gg 1$ is satisfied. In this case the extremely difficult task of averaging of Eqs.(5),(7), derived for a given trajectory, over all possible trajectories of electrons in a crystal simplifies radically. In fact, if $\varrho_c \gg 1$ then in the range where trajectories are essentially non-rectilinear ($\vartheta_0 \leq \vartheta_c$, $v_\perp \sim \vartheta_c$) the mechanism of photon emission is of the magnetic bremsstrahlung nature and the characteristics of radiation can be expressed in terms of local parameters of motion. Then the averaging procedure can be carried out simply if one knows the distribution function in the transverse phase space $dN(\varrho, \mathbf{v}_\perp)$, which for a thin crystal is defined directly by the initial conditions of incidence of particle on a crystal.

Substituting Eq.(5) into Eq.(1) we find after integration by parts of terms $\mathbf{n}\mathbf{v}_{1,2}$ ($\mathbf{n}\mathbf{v}_{1,2} \rightarrow 1$) the general expression for photon emission probability (or probability of creation of longitudinally polarized positron)

$$\begin{aligned} dw_\xi &= \sigma \frac{\alpha m^2}{8\pi^2} \frac{d\Gamma}{\varepsilon \varepsilon'} \int \frac{dN}{N} \int e^{i\sigma A} \left[\varphi_1(\xi) - \frac{\sigma}{4} \varphi_2(\xi) \gamma^2 (\mathbf{v}_1 - \mathbf{v}_2)^2 \right] dt_1 dt_2, \\ A &= \frac{\omega \varepsilon}{2\varepsilon'} \int_{t_1}^{t_2} \left[\frac{1}{\gamma^2} + (\mathbf{n} - \mathbf{v}(t))^2 \right] dt, \\ \varphi_1(\xi) &= 1 + \xi \frac{\omega}{\varepsilon}, \quad \varphi_2(\xi) = (1 + \xi) \frac{\varepsilon}{\varepsilon'} + (1 - \xi) \frac{\varepsilon'}{\varepsilon}. \end{aligned} \quad (9)$$

where $\alpha = e^2 = 1/137$, the vector \mathbf{n} is defined in Eq.(1), the helicity of emitted photon ξ is defined in Eq.(5), $\sigma = -1$, $d\Gamma = d^3k$ for radiation and $\sigma = 1$, $d\Gamma = d^3p$ for pair creation and for pair creation one have to put $\int dN/N = 1$.

The circular polarization of radiation is defined by Stoke's parameter $\xi^{(2)}$:

$$\xi^{(2)} = \Lambda(\boldsymbol{\zeta}\mathbf{v}), \quad \Lambda = \frac{dw_+ - dw_-}{dw_+ + dw_-}, \quad (10)$$

where the quantity $(\boldsymbol{\zeta}\mathbf{v})$ defines the longitudinal polarization of the initial electrons, dw_+ and dw_- is the probability of photon emission for $\xi=+1$ and $\xi=-1$ correspondingly. In the limiting case $\omega \ll \varepsilon$ one has $\varphi_2(\xi) \simeq 2(1 + \xi\omega/\varepsilon) = 2\varphi_1(\xi)$. So the expression for the probability dw_ξ contains the dependence on ξ as a common factor $\varphi_1(\xi)$ only. Substituting in Eq.(10) one obtains the universal result independent of a particular mechanism of radiation $\xi^{(2)} = \omega(\boldsymbol{\zeta}\mathbf{v})/\varepsilon$.

The periodic crystal potential $U(\mathbf{r})$ can be presented as the Fourier series (see e.g. [4], Sec.8)

$$U(\mathbf{r}) = \sum_{\mathbf{q}} G(\mathbf{q}) e^{-i\mathbf{q}\mathbf{r}}, \quad (11)$$

where $\mathbf{q} = 2\pi(n_1, n_2, n_3)/l$; l is the lattice constant. The particle velocity can be presented in a form $\mathbf{v}(t) = \mathbf{v}_0 + \Delta\mathbf{v}(t)$, where \mathbf{v}_0 is the average velocity. If $\vartheta_0 \gg \vartheta_c$, we find $\Delta\mathbf{v}(t)$ using the rectilinear trajectory approximation

$$\Delta\mathbf{v}(t) = -\frac{1}{\varepsilon} \sum_{\mathbf{q}} \frac{G(\mathbf{q})}{q_{\parallel}} \mathbf{q}_{\perp} \exp[-i(q_{\parallel}t + \mathbf{q}\mathbf{r})], \quad (12)$$

where $q_{\parallel} = (\mathbf{q}\mathbf{v}_0)$, $\mathbf{q}_{\perp} = \mathbf{q} - \mathbf{v}_0(\mathbf{q}\mathbf{v}_0)$. Substituting Eq.(12) into Eq.(9), performing the integration over $\mathbf{u} = \mathbf{n} - \mathbf{v}_0$ ($d^3k = \omega^2 d\omega d\mathbf{u}$, $d^3p = \varepsilon^2 d\varepsilon d\mathbf{u}$) and passing to the variables t, τ : $t_1 = t - \tau$, $t_2 = t + \tau$, we obtain after simple calculations the general expression for the probability of photon emission (or probability of pair creation) for polarized case valid for any angle of incidence ϑ_0

$$dw_{\xi} = \frac{i\alpha m^2 d\Gamma_1}{4\pi} \int \frac{dN}{N} \int \frac{d\tau}{\tau + i\sigma 0} \left[\varphi_1(\xi) + \sigma \varphi_2(\xi) \right. \\ \left. \times \left(\sum_{\mathbf{q}} \frac{G(\mathbf{q})}{mq_{\parallel}} \mathbf{q}_{\perp} \sin(q_{\parallel}\tau) e^{-i\mathbf{q}\mathbf{r}} \right)^2 \right] e^{i\sigma A_2}, \quad (13)$$

where

$$A_2 = \frac{m^2 \omega \tau}{\varepsilon \varepsilon'} \left[1 + \sum_{\mathbf{q}, \mathbf{q}'} \frac{G(\mathbf{q}) G(\mathbf{q}')}{m^2 q_{\parallel} q'_{\parallel}} (\mathbf{q}_{\perp} \mathbf{q}'_{\perp}) \Psi(q_{\parallel}, q'_{\parallel}, \tau) \exp[-i(\mathbf{q} + \mathbf{q}')\mathbf{r}] \right] \\ \Psi(q_{\parallel}, q'_{\parallel}, \tau) = \frac{\sin(q_{\parallel} + q'_{\parallel})\tau}{(q_{\parallel} + q'_{\parallel})\tau} - \frac{\sin(q_{\parallel}\tau)}{q_{\parallel}\tau} \frac{\sin(q'_{\parallel}\tau)}{q'_{\parallel}\tau}, \quad (14)$$

where $d\Gamma_1 = \omega d\omega / \varepsilon^2 (\varepsilon d\varepsilon / \omega^2)$ for radiation (pair creation).

3 Radiation and pair creation for $\vartheta_0 \ll V_0/m$ (constant field limit)

The behavior of probability dw_{ξ} Eq.(13) for various entry angles and energies is determined by the dependence of the phase A_2 on these parameters given Eq.(14). In the axial case for $\vartheta_0 \ll V_0/m \equiv \vartheta_v$ the main contribution to A_2 give vectors \mathbf{q} lying in the plane transverse to the axis [4] and the problem becomes two-dimensional with the potential $U(\boldsymbol{\rho})$. Performing the calculation for axially symmetric $U = U(\boldsymbol{\rho}^2)$ we obtain

$$dw_{\xi}^F(\omega) = \frac{\alpha m^2 \omega d\Gamma_1}{2\sqrt{3}\pi} \int_0^{x_0} \frac{dx}{x_0} \left\{ DR_0(\lambda) - \frac{1}{6} \left(\frac{m\vartheta_0}{V_0} \right)^2 \left[\frac{xg'' + 2g'}{xg^3} R_1(\lambda) \right. \right.$$

$$\left. -\frac{\lambda}{20g^4x^2} \left(2x^2g'^2 + g^2 + 14gg'x + 6x^2gg'' \right) R_2(\lambda) \right] \Bigg\}, \quad (15)$$

where

$$\begin{aligned} R_0(\lambda) &= \varphi_2(\xi)K_{2/3}(\lambda) + \varphi_1(\xi) \int_{\lambda}^{\infty} K_{1/3}(y)dy, \quad R_1(\lambda) = \varphi_2(\xi) \left[(K_{2/3}(\lambda) \right. \\ &\quad \left. - \frac{2}{3\lambda}K_{1/3}(\lambda) \right], \quad R_2(\lambda) = \varphi_1(\xi) \left(K_{1/3}(\lambda) - \frac{4}{3\lambda}K_{2/3}(\lambda) \right) \\ &\quad - \varphi_2(\xi) \left(\frac{4}{\lambda}K_{2/3}(\lambda) - \left(1 + \frac{16}{9\lambda^2} \right) K_{1/3}(\lambda) \right), \end{aligned} \quad (16)$$

here $D = \int dN/N (D = 1)$ for the radiation (pair creation), $K_{\nu}(\lambda)$ is the modified Bessel function (McDonald's function), we have adopted a new variable $x = \varrho^2/a_s^2$, $x \leq x_0$, $x_0^{-1} = \pi a_s^2 dn_a = \pi a_s^2/s$, a_s is the effective screening radius of the potential of the string, n_a is the density of atoms in a crystal, d is the average distance between atoms of a chain forming the axis. The term in Eq.(15) with $R_0(\lambda)$ represent the spectral probability in the constant field limit. The other terms are the correction proportional ϑ_0^2 arising due to nongomogeneity of field in crystal. The notation $U'(x) = V_0g(x)$ is used in Eq.(15) and

$$\lambda = \frac{m^3 a_s \omega}{3\varepsilon\varepsilon' V_0 g(x) \sqrt{x}} = \frac{u}{3\chi_s g(x) \sqrt{x}} = \frac{2\omega^2}{3\varepsilon\varepsilon'} \frac{\sqrt{\eta}}{\kappa_s \psi(x)}, \quad \psi(x) = 2\sqrt{\eta x} g(x), \quad (17)$$

here $\kappa_s = V_0\omega/(m^3 a_s)$, $\chi_s = V_0\varepsilon/(m^3 a_s)$ are a typical values of corresponding parameters in crystal. For specific calculation we use the following for the potential of axis:

$$U(x) = V_0 \left[\ln \left(1 + \frac{1}{x + \eta} \right) - \ln \left(1 + \frac{1}{x_0 + \eta} \right) \right]. \quad (18)$$

For estimates one can put $V_0 \simeq Ze^2/d$, $\eta \simeq 2u_1^2/a_s^2$, where Z is the charge of the nucleus, u_1 is the amplitude of thermal vibrations, but actually the parameter of potential were determined by means of a fitting procedure using the potential Eq.(11) (table of parameters for different crystals is given in Sec.9 of [4]). For this potential

$$g(x) = \frac{1}{x + \eta} - \frac{1}{x + \eta + 1} = \frac{1}{(x + \eta)(x + \eta + 1)} \quad (19)$$

The result of calculation of spectral intensity ($dI_{\xi} = \omega dw_{\xi}$), $\varepsilon \frac{dI_{\xi}^F(\omega)}{d\omega}$ in tungsten crystal, axis $\langle 111 \rangle$, $T = 293$ K) is given in Fig.1.

In Fig.2 the circular polarization $\xi^{(2)}$ of radiation is plotted versus ω/ε for the same crystal. This curve is true for both energies: $\varepsilon = 250$ GeV and $\varepsilon = 1$ TeV.

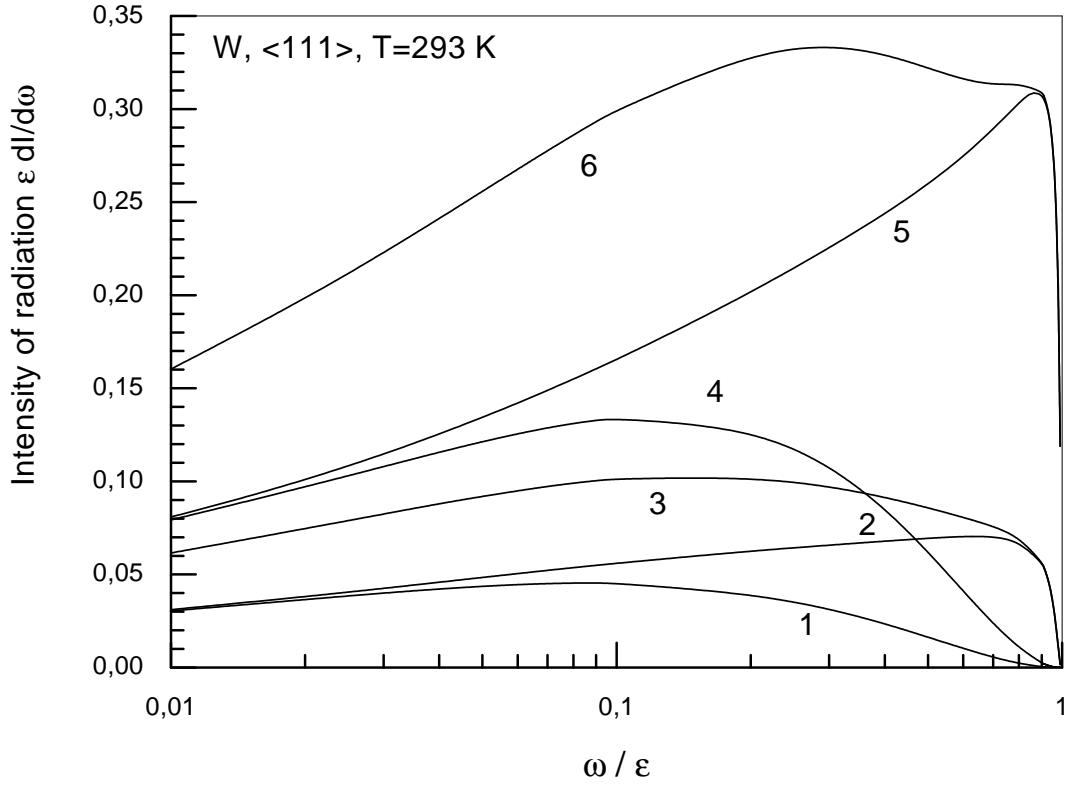


Figure 1: Spectral intensity of radiation in units αm^2 vs ω/ε . The curves 1, 4 are for $\xi = -1$, the curves 2, 5 are for $\xi = 1$, the curves 3, 6 are the sum of previous contributions (the probability for unpolarized particles). The curves 1, 2, 3 are for the initial electron energy $\varepsilon=250$ GeV and the curves 4, 5, 6 are for the initial electron energy $\varepsilon=1$ TeV.

Actually this means that it is valid for any energy in high-energy region. At $\omega/\varepsilon=0.8$ one has $\xi^{(2)}=0.94$ and at $\omega/\varepsilon=0.9$ one has $\xi^{(2)}=0.99$.

Now we pass to a pair creation by a photon. In the limit $\kappa_s \ll 1$ one can substitute the asymptotic of functions $K_\nu(\lambda)$ at $\lambda \gg 1$ in the term with $R_0(\lambda)$ in Eq.(15). After this one obtains using the Laplace method in integration over the coordinate x

$$\frac{dW_\xi^F}{dy} = \frac{\sqrt{3}\alpha V_0 (y(1+\xi) + (1-y)^2)}{2mx_0a_s} \left[\frac{\psi^3(x_m)}{4\eta|\psi''(x_m)|} \right]^{1/2} e^{-\frac{2}{3y(1-y)\kappa_m}}, \quad (20)$$

where $\kappa_m = \kappa_s \psi(x_m)/\sqrt{\eta}$. So, at low energies the pair creation probability is suppressed. With photon energy increase the probability increases also and at some energy $\omega \simeq \omega_t$ it becomes equal to the standard Bethe-Maximon W_{BM} probability in the considered medium, e.g. for W, T=293 K, axis $<111>$ $\omega_t = 22$ GeV. For high energies the probability of pair creation may be much higher

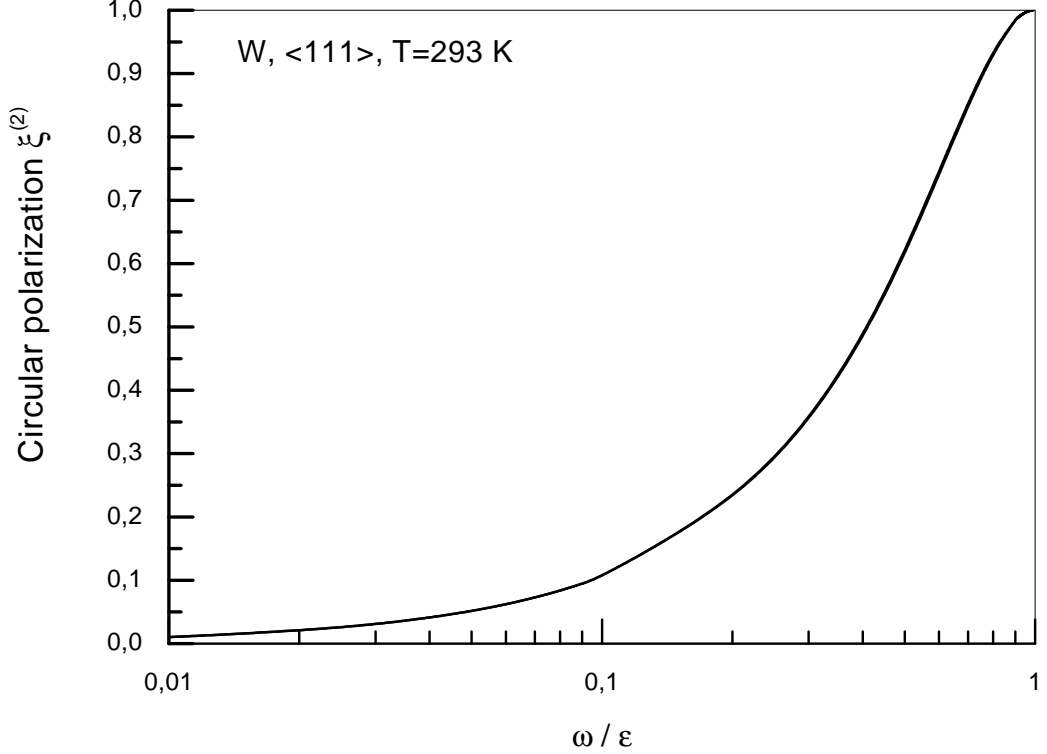


Figure 2: The circular polarization $\xi^{(2)}$ of radiation (for $(\zeta \mathbf{v})=1$) vs ω/ε for the tungsten crystal, axis $\langle 111 \rangle$, $T = 293 \text{ K}$. The curve is valid for both energies: $\varepsilon=250 \text{ GeV}$ and $\varepsilon=1 \text{ TeV}$.

than W_{BM} (see [4]).

This is seen in Fig.3 where the spectral probability of pair creation dw_ξ/dy in tungsten, $T=293 \text{ K}$, axis $\langle 111 \rangle$ is given. Near the end of spectrum the process with $\xi = 1$ dominates. The sum of curves at the indicated energy gives unpolarized case. For the integral (over y) probability the longitudinal polarization of positron is $\zeta = 2/3$.

4 Modified theory of coherent bremsstrahlung and pair creation

The estimates of double sum in the phase A_2 made at the beginning of previous section: $\sim (\vartheta_v/\vartheta_0)^2 \Psi$ remain valid also for $\vartheta_0 \geq \vartheta_v$, except that now the factor in the double sum is $(\vartheta_v/\vartheta_0)^2 \leq 1$, so that the values $|q_{\parallel}\tau| \sim 1$ contribute. We consider first the limiting case $\vartheta_0 \gg \vartheta_v$, then this factor is small and $\exp(-iA_2)$ can be expanded accordingly. After integration over coordinate \mathbf{r} we obtain

$$dw_\xi^{coh} = \frac{\alpha d\Gamma_1}{8} \sum_{\mathbf{q}} |G(\mathbf{q})|^2 \frac{\mathbf{q}_\perp^2}{q_\parallel^2} \left[\varphi_2(\xi) + \sigma \varphi_1(\xi) \frac{2m^2\omega}{\varepsilon\varepsilon'q_\parallel^2} \left(|q_\parallel| - \frac{m^2\omega}{2\varepsilon\varepsilon'} \right) \right]$$

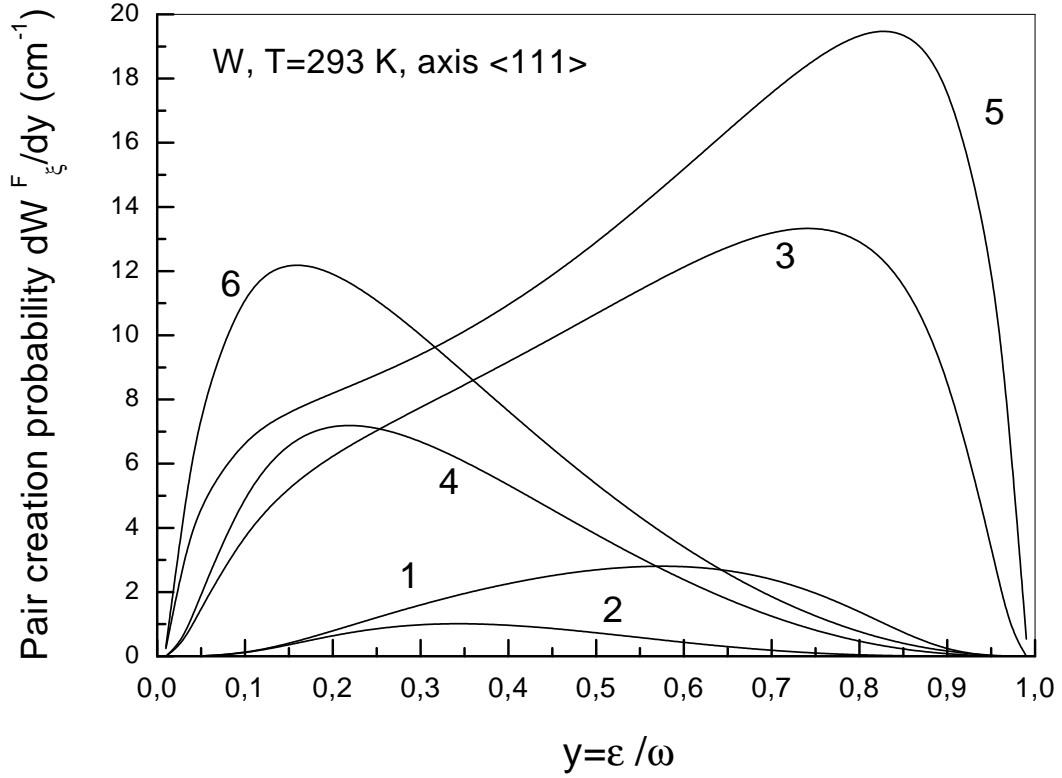


Figure 3: The spectral probability of pair creation dw_{ξ}^F/dy , the curves 1 and 2 are for energy $\varepsilon=22$ GeV, the curves 3 and 4 are for energy $\varepsilon=100$ GeV, the curves 5 and 6 are for energy $\varepsilon=250$ GeV. The curves 1, 3 and 5 are for $\xi = 1$, and the curves 2, 4 and 6 are for $\xi = -1$.

$$\times \vartheta \left(|q_{\parallel}| - \frac{m^2 \omega}{2\varepsilon \varepsilon'} \right). \quad (21)$$

For unpolarized electrons (the sum of cotributions with $\xi = 1$ and $\xi = -1$) Eq.(21) coincides with the result of standard theory of coherent bremsstrahlung (CBS), or coherent pair creation see e.g. [9].

In the case $\chi_s \gg 1$ (χ_s is defined in Eq.(17)), one can obtain more general expression for the spectral probability:

$$dw_{\xi}^{mcoh} = \frac{\alpha d\Gamma_1}{8} \sum_{\mathbf{q}} |G(\mathbf{q})|^2 \frac{\mathbf{q}_{\perp}^2}{q_{\parallel}^2} \left[\varphi_2(\xi) + \sigma \varphi_1(\xi) \frac{2m^2 \omega}{\varepsilon \varepsilon' q_{\parallel}^2} \left(|q_{\parallel}| - \frac{m_*^2 \omega}{2\varepsilon \varepsilon'} \right) \right] \times \vartheta \left(|q_{\parallel}| - \frac{m_*^2 \omega}{2\varepsilon \varepsilon'} \right), \quad m_*^2 = m^2 \left(1 + \frac{\rho}{2} \right), \quad \frac{\rho}{2} = \sum_{\mathbf{q}, q_{\parallel} \neq 0} \frac{|G(\mathbf{q})|^2}{m^2 q_{\parallel}^2} \mathbf{q}_{\perp}^2 \quad (22)$$

The spectral probabilities Eqs.(21) and (22) can be much higher than the Bethe-Maximon bremsstrahlung probability W_{BM} for small angles of incidence

ϑ_0 with respect to selected axis. For the case $\vartheta_0 \ll 1$ the quantity q_{\parallel} can be represented as

$$q_{\parallel} \simeq \frac{2\pi}{d}n + \mathbf{q}_{\perp} \mathbf{v}_{0,\perp} \quad (23)$$

In the extreme limit, when the parameter $\lambda \equiv 2\varepsilon|q_{\parallel}|_{\min}/m^2 \sim \varepsilon\vartheta_0/m^2a_s \gg 1$, the maximum of probability of coherent bremsstrahlung is attained at such values of ϑ_0 where the standard theory of coherent bremsstrahlung becomes invalid. Bearing in mind that if $\lambda \gg 1$ and $\vartheta_0 \sim V_0/m$, then $\chi_s \sim \lambda \gg 1$, we can conveniently use a modified theory of coherent bremsstrahlung.

The direction of transverse components of particle's velocity in Eq.(23) can be selected in a such way, that the spectral probability described by Eq.(22) has a sharp maximum near the end of spectrum at $\omega = 2\varepsilon\lambda(2 + 2\lambda + \varrho)^{-1} \simeq \varepsilon$ with relatively small (in terms of λ^{-1}) width $\Delta\omega \sim \varepsilon(1 + \varrho/2)/\lambda = m^2(1 + \varrho/2)/2|q_{\parallel}|_{\min}$:

$$(dw_{\xi})_{\max} = \frac{\alpha\varepsilon d\Gamma_1\varrho|q_{\parallel}|_{\min}}{4(2 + \varrho)} \left(1 + \xi + \frac{1 - \xi}{(1 + u_m)^2}\right), \quad u_m = \frac{2\lambda}{2 + \varrho}. \quad (24)$$

It is seen that in the maximum of spectral distribution the radiation probability with opposite helicity ($\xi = -1$) is suppressed as $1/(1 + u_m)^2$. At $u > u_m$ one have to take into account the next harmonics of particle acceleration. In this region of spectrum the suppression of radiation probability with opposite helicity is more strong, so the emitted photons have nearly complete circular polarization.

Comparing Eq.(24) with W_{BM} for $\varepsilon' \ll \varepsilon$ we find that for the same circular polarization ($\xi^{(2)} \simeq (\boldsymbol{\zeta} \mathbf{v})$) in the particular case $\varrho = 1$ the magnitude of spectral probability in Eq.(24) is about $\chi_s(\varepsilon_e)$ times larger than W_{BM} . For tungsten $\chi_s(\varepsilon_e) = 78$. From above analysis follows that under mentioned conditions the considered mechanism of emission of photons with circular polarization is especially effective because there is gain both in monochromaticity of radiation and total yield of polarized photons near hard end of spectrum.

5 Conclusions

At high energy the radiation from longitudinally polarized electrons in oriented crystals is circularly polarized and $\xi^{(2)} \rightarrow 1$ near the end of spectrum. This is true in magnetic bremsstrahlung limit $\vartheta_0 \ll V_0/m$ as well as in coherent bremsstrahlung region $\vartheta_0 > V_0/m$. This is particular case of helicity transfer.

In crossing channel: production of electron-positron pair with longitudinally polarized particles by the circularly polarized photon in an oriented crystal the same phenomenon of helicity transfer takes place in the case when the final particle takes away nearly all energy of the photon.

So, the oriented crystal is a very effective device for helicity transfer from an electron to photon and back from a photon to electron or positron. Near the end of spectrum this is nearly 100% effect.

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